Sesión Especial 4

Análisis Geométrico Convexo

Organizadores

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Descripción

El origen del Análisis Geométrico Convexo se puede situar en la creciente activa interacción entre la Geometría clásica (convexa) y el Análisis (convexo) asintótico. Durante las últimas dos décadas, la geometría integral de los cuerpos convexos ha experimentado una revitalización impulsada por la introducción de métodos, resultados y, sobre todo, nuevos puntos de vista provenientes de otras ramas de las matemáticas como son la teoría de la probabilidad, el análisis armónico, la geometría de espacios normados finito dimensionales, la geometría integral y la geometría discreta.

El objetivo principal de esta sesión especial es proporcionar los medios óptimos para la actividad investigadora en el ámbito del Análisis Geométrico Convexo y áreas afines. Para ello reuniremos expertos, miembros tanto de universidades españolas como de universidades extranjeras, de áreas como Análisis Geométrico, Geometría Convexa, Geometría Integral y Geometría Discreta. Esta sesión está también especialmente enfocada a fomentar la cooperación entre investigadores jóvenes e investigadores senior, para potenciar el avance de nuevos y recientes resultados en los mencionados campos de investigación.

Programa

JUEVES, 7 de febrero (mañana)

11:30 – 12:00	María A. Hernández Cifre (Universidad de Murcia) *On a characterization of (dual) mixed volumes*
12:00 – 12:30	Rafael Villa (Universidad de Sevilla) *Functional version of local Loomis-Whitney type inequality*
12:30 – 13:00	Bernardo González Merino (University of Graz) *On Hermite-Hadamard and Rogers-Shephard inequalities*
13:00 – 13:30	Antonio Cañete (Universidad de Sevilla) *Minimizing the maximum relative diameter*

VIERNES, 8 de febrero (mañana)

9:00 – 9:30	Gil Solanes (Universidad Autónoma de Barcelona) *Lipschitz-Killing valuations in pseudo-riemannian manifolds*
9:30 – 10:00	César Rosales (Universidad de Granada) *Classification results for CMC surfaces in convex domains with weights*
10:00 – 10:30	Jesús Yepes Nicolás (Universidad de Murcia) *On discrete Borell-Brascamp-Lieb inequalities*
10:30 – 11:00	Carlos Hugo Jiménez (PUC-Rio) *Some functional affine isoperimetric and other related inequalities*

Minimizing the maximum relative diameter

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Abstract. Given a planar convex body C, a *bisection* of C is a decomposition of C into two connected subsets $\{C_1, C_2\}$. For each bisection P of C, the *maximum relative diameter* is defined by

$$d_M(P, C) = \text{máx}\{D(C_1), D(C_2)\},$$

where $D(C_i)$ denotes the Euclidean diameter of C_i, $i = 1, 2$. It is clear that each different bisection of C will give a different value for d_M.
We are interested in the bisections minimizing the functional d_M, as well as in the corresponding minimal value

$$D_B(C) = \text{mín}\{d_M(P, C) : P \text{ is a bisection of } C\}.$$

In this talk we shall describe some properties regarding the minimizing bisections and, for each planar convex body C, we shall obtain an isodiametric inequality associated with this problem:

$$\frac{A(C)}{D_B(C)^2} \leq \alpha,$$

where $A(C)$ denotes the area of C and α is a positive constant. We shall also characterize the set providing the equality above.

Joint work with Bernardo González Merino (Universidad de Sevilla)

On a characterization of (dual) mixed volumes

María Ángeles Hernández Cifre

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Abstract. The main aim of this talk will be to present a beautiful classical problem on mixed volumes which is still open, as well as its version in the dual setting. Given r convex bodies in the n-dimensional Euclidean space, there are $N = \binom{n+r-1}{n}$ mixed volumes associated to them. Then, a set of inequalities is said to be a full set if given N (non-negative) numbers satisfying the inequalities, they arise as the mixed volumes of r convex bodies. In 1960, Shephard investigated whether the known inequalities (Aleksandrov-Fenchel and some determinantal inequalities) are a full set, and solved it when two convex bodies come into play. Except for a few particular cases, for arbitrary r, the problem is still open.
Next we will consider the corresponding question in the dual Brunn-Minkowski theory, i.e., to look for necessary and sufficient conditions for $n+1$ positive real numbers to be the dual mixed volumes of two star bodies in the n-dimensional Euclidean space.

Joint work with David Alonso-Gutiérrez and Martin Henk

Some functional affine isoperimetric and other related inequalities

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Abstract. In this talk we will review some recent extensions and generalizations of affine isoperimetric type inequalities like Blaschke Santaló and Busemann-Petty centroid. Our methods are arguably more direct than in previous approaches but still allowing us to recover equality cases.

Joint work with J. Haddad, L. Silva and M. Montenegro

On Hermite-Hadamard and Rogers-Shephard inequalities

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Abstract. The original Hermite-Hadamard inequality (1881) states that if $f : \mathbb{R} \to \mathbb{R}$ is a concave function, then

$$\frac{1}{b-a}\int_a^b f(x)dx \leq f\left(\frac{a+b}{2}\right) \tag{1}$$

for any $a < b$. In this talk we will derive some new extensions of (1) in $\mathbb{R}^n$ replacing $f(x)$ by $f(x)^m$ for some $m \in \mathbb{N}$. As an application to these new inequalities, we will derive Rogers-Shephard type inequalities. The latter inequalities relate the volume of a convex compact set K in $\mathbb{R}^n$ to the volumes of some of its sections and projections with respect to some linear subspaces, and some of them is considered as the reverse to the Brunn-Minkowski inequality.

Classification results for CMC surfaces in convex domains with weights

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Abstract. In a Euclidean domain we consider a positive function which is used to weight the Hausdorff measures. Then, it is possible to study the surfaces in the domain which are critical for the weighted area, possibly for fixed weighted volume. As in the unweighted case, these surfaces satisfy some kind of constant mean curvature (CMC) condition, which involves the Euclidean mean curvature of the surface and the weight. In this talk we will show some results describing CMC surfaces in convex domains with certain weights. In particular, we will prove a Bernstein-type theorem stating that the only entire CMC graphs for some Gauss-like product weights are Euclidean planes.

Lipschitz-Killing valuations in pseudo-riemannian manifolds

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Abstract. The Lipschitz-Killing invariants discovered by H. Weyl are among the most fundamental quantities that can be assigned to a compact riemannian manifold. Besides Weyl's tube formula, they appear in seemingly unrelated situations such as the kinematic formula of Blaschke-Santaló-Chern and the heat kernel of differential forms.
Notably, the Lipschitz-Killing invariants can also be defined on sufficiently nice compact subsets of any riemannian manifold. In this form, they belong to a class of functionals called (smooth) valuations, and they provide a natural extension of the classical quermassintegrals of euclidean convex bodies.
In the talk we will present a joint work with Andreas Bernig and Dmitry Faifman where the Lipschitz-Killing valuations are generalized to the setting of pseudo-riemannian manifolds.

Joint work with Andreas Bernig and Dmitry Faifman

Functional version of local Loomis-Whitney type inequality

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Abstract. The classical Loomis-Whitney inequality [Villa-3] states that for any n-dimensional compact and convex set K and hyperplanes H_k, $k = 1, \dots, n$, with $H_k^{\perp} \subset H_l$ for $l \neq k$, then

$$\mathrm{vol}_n(K)^{n-1} \leq \prod_{k=1}^{n} \mathrm{vol}_{n-1}(P_{H_k}K). \tag{2}$$

Equality holds above if and only if K is a box with facets parallel to each H_k. If, rather than considering n subspaces, we restrict to 2 of them, then we arrive onto the so-called *local Loomis-Whitney type* inequalities. An exhaustive study of those inequalities is done in [Villa-3].
In this talk we present a sharp local Loomis-Whitney type inequality. More precisely, let K be an n-dimensional compact and convex set, E (resp. H) be an i-(resp. j)-dimensional subspace, be such that $i, j \in \{2, \dots, n-1\}$, $i + j \geq n + 1$, and $E^{\perp} \subset H$. Denote $k = i + j - n$, so that $1 \leq k \leq n - 2$. Then

$$\mathrm{vol}_k(P_{E\cap H}K)\mathrm{vol}_n(K) \leq \frac{\binom{i}{k}\binom{j}{k}}{\binom{n}{k}} \mathrm{vol}_i(P_E K)\mathrm{vol}_j(P_H K). \tag{3}$$

We also give the cases in which equality occurs.
We compare this result with the known "local" results. For the proof, we borrow from [Villa-3] the idea of using Berwald's inequality [Villa-2], but a more general version of the same author.
We also find the functional analogue of (3). Namely, let f be a log-concave function on $\mathbb{R}^n$, and let E (resp. H) be an i-(resp. j)-dimensional linear subspace be such that $i, j \in \{2, \dots, n-1\}$, $i + j \geq n + 1$, and $E^{\perp} \subset H$. Denote $k = i + j - n$, so that $1 \leq k \leq n - 2$. Then

$$\int_{E\cap H} P_{E\cap H} f(w)dw \int_{\mathbb{R}^n} f(z)dz \leq \binom{n-k}{n-i} \int_E P_E f(x)dx \int_H P_H f(y)dy. \tag{4}$$

In order to prove (4), we will need to prove a suitable version of Berwald's inequality. The talk is based on the results contained in [Villa-1].

Referencias

[Villa-1] D. Alonso-Gutiérrez, S. Artstein-Avidan, B. González-Merino, C. H. Jiménez, R. Villa, *Rogers-Shephard and local Loomis-Whitney type inequalities*, arXiv:1706.01499 (2017).

[Villa-2] L. Berwald, *Verallgemeinerung eines Mittelwetsatzes von J. Favard, für positive konkave Funktionen*, Acta Math. **79** (1947), pp. 17–37.

[Villa-3] S. Brazitikos, A. Giannopoulos, D. M. Liakopoulos, *Uniform cover inequalities for the volume of coordinate sections and projections of convex bodies*, arXiv:1606.03779 (2016).

[Villa-3] L. H. Loomis, H. Whitney, *An inequality related to the isoperimetric inequality*, Bull. Amer. Math. Soc. **55** (1949), 961–962.

Joint work with David Alonso-Gutiérrez, Shiri Artstein-Avidan, Bernardo González Merino and C. Hugo Jiménez

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On discrete Borell-Brascamp-Lieb inequalities

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Abstract. If $f, g, h : \mathbb{R}^n \longrightarrow \mathbb{R}_{\geq 0}$ are non-negative measurable functions such that $h(x+y)$ is greater than or equal to the p-sum of $f(x)$ and $g(y)$, where $-1/n \leq p \leq \infty$, $p \neq 0$, then the classical Borell-Brascamp-Lieb inequality asserts that the integral of h is not smaller than the q-sum of the integrals of f and g, for $q = p/(np+1)$.

In this talk we will show a discrete analog of the above-mentioned result for the sum over finite subsets of the integer lattice $\mathbb{Z}^n$: under the same assumption as before, for $A, B \subset \mathbb{Z}^n$, then $\sum_{A+B} h \geq [(\sum_{\mathrm{r}f(A)} f)^q + (\sum_B g)^q]^{1/q}$, where $\mathrm{r}f(A)$ is obtained by removing points from A in a particular way, and depending on f. In particular, different Brunn-Minkowski type inequalities are obtained when considering certain discrete measures on $\mathbb{R}^n$.

We will also show that the classical Borell-Brascamp-Lieb inequality for Riemann integrable functions can be derived as a consequence of this new discrete version.

Joint work with David Iglesias López (Universidad de Murcia)
